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Optimal Selling Mechanisms under Imperfect Commitment*

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Abstract

This paper studies the optimal mechanisms for a seller with imperfect commitment who puts up for sale one individual unit per period to a single buyer in a dynamic game. The buyer's willingness to pay remains constant over time and is his private information. In this setting, the seller cannot achieve greater payoffs than those obtained by posting a price in each period. However, price posting is not optimal if the buyer is sufficiently impatient relative to the seller. It is also proved that a mechanism à la Goethe (see Moldovanu and Tieztel 1998) is almost optimal.

Keywords: asymmetric information, imperfect commitment, dynamics, mechanism design, non-optimality of posting prices.

JEL codes: D82

1 Introduction

In 1797, Goethe was in the process of trying to sell his most recent work, the epic poem Hermann and Dorothea. However, he was concerned about the information asymmetry between him and the publisher with respect to the publisher's valuation of his work.[1] Goethe decided to propose the following selling mechanism: each one (Goethe and the publisher) would send a sealed note with their demanded price to a lawyer; the sale would take place at Goethe's price only in the case that the publisher's demanded price was higher than or equal to Goethe's demanded price.[2] With this mechanism, Goethe wanted to learn something about the publisher's valuation and obtain some advantage in future transactions.

Goethe's story illustrates a common situation in the market place. A seller wants to sell something to a buyer whose willingness to pay is private. The seller may use information from past sales to the same buyer to infer his willingness to pay and to implement pricing schemes that better discriminate among consumers. However, from a theoretical point of view, this is not obvious because consumers (the publisher in Goethe's case) may have incentives to act strategically to mislead the learning process of the seller.

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[1] See Moldovanu, B. and Tieztel, M. (1998) for the complete story.

[2] As Moldovanu and Tietzel (1998) pointed this is a second-price auction in which the sealed reserve price of the seller has the effect of a second bidder.

For instance, Hart and Tirole (1988) argue that a monopolist that sells a perishable unit in each period and has full commitment power at the beginning of the game finds it optimal to commit to ignoring all the information that she learns along the equilibrium path about the type of the buyer. Then, the buyer has no incentives to lie during one period to manipulate the seller's belief. However, full commitment is a very extreme assumption and it is possible to find many situations in which a short-term commitment relationships fits better.[3]

There is extensive literature related to bargaining under conditions of asymmetric information where only one party has the right to make offers. Most of them study the case of durable goods, in which the game finishes when a buyer accepts an offer.[4] On the other hand, Hart and Tirole (1988) and Schmidt (1992) study the case of repeated bargaining where a player with bargaining power trades a service or perishable good in every period with non-anonymous and sufficiently patient agents.[5] All these previous articles restrict the monopolist's strategy to a sequence of posted prices. Nothing guarantees that this mechanism is the optimal one. It is natural to ask if the monopolist has a better selling mechanism to maximize her benefits.

The purpose of this paper is to study the conditions under which price posting may be an optimal selling mechanism. In particular, this question is studied in a framework in which a seller commits to use a selling mechanism for the current period, but not for future ones. It is considered a two-period model with one seller (she) and one buyer (he). The buyer has two possible valuations for the good which are his private information. In every period the seller has one perishable good to sell, which is produced at zero cost.

The main result is that the seller cannot achieve greater expected payoff than the one obtained by posting a price in each period. However, price posting is not optimal if the buyer is sufficiently impatient relative to the seller. In this last case, there are intertemporal gains from trade that the seller exploits when using a non-price posting selling mechanism. As consequence, the selling model and the renting model for a durable good are not any more equivalent when one does not restrict attention to price posting mechanism.

The paper also shows that it is possible to rationalize the mechanism used by Goethe. Although Moldovanu and Tieztel (1998) shows that Goethe's mechanism is optimal in an static framework, the description of the story fits better with a dynamic framework. In this framework, Goethe's mechanism is almost optimal if one assumes that the publisher is relatively impatient with respect to Goethe. This seems a reasonable assumption since publishers could not count on dealing with Goethe in the future with any type of certainty.

Skreta (2006) has shown that posting a price is the optimal selling mechanism when a monopolist with a short-term commitment has a durable good to sell to a single buyer that the monopolist addresses repeatedly. This paper looks for the optimal selling mechanism when such a monopolist rents that durable good or, instead, has a perishable good or a service to sell. Technically, there is a crucial difference. In Skreta (2006), the game finishes when the buyer buys the good, but this is not the case in the model developed below. Her procedure is not directly applicable to it because she sustains her analysis on the fact that the only non-trivial continuation value arises when the good is not sold. In the model proposed in this paper, the buyer has to take into account how his future surplus is going to be affected in case of buying and in case of rejecting the good, i.e. there are two continuation values.

The model is solved using a dynamic mechanism design approach following the procedure proposed in Bester and Strausz (2001). In that article, they provide a modified version of the revelation

[3] For some real world examples about the inability of the principal to commit see Laffont and Tirole (1988), Laffont and Tirole (1993) or McAfee and Vincent (1997).

[4] See for example Fudenberg, Levine and Tirole (1985), or Sobel and Takahashi (1983).

[5] Hart and Tirole study both cases: the durable good case and the case in which the monopolist decides to rent it.

principle where the seller has imperfect commitment.

The paper is organized as follows. Section 2 provides a general setup of the problem and reviews the Bester and Strausz (2001) revelation principle for this type of environment. Section 3 analyzes the problem and gives a characterization of the optimal selling mechanism. Section 4 proves that this result does not hold when players have different discount factors, discusses its implications and shows that the mechanism proposed by Goethe can be interpreted in this direction. Finally, Section 5 concludes. All proofs are relegated to the Appendix.

2 General Setup

Next, it is proposed a dynamic problem that follows the framework in Bester and Strausz (2001).[6] The problem is solved by recursive methods as they suggest. Therefore, this section directly states a dynamic problem as a sequence of static problems.

Consider a two-period game with $r = \{1, 2\}$, where r is the number of periods remaining at the beginning of the current period. There is one risk-neutral seller (the principal) and one risk-neutral buyer (the agent) facing each other repeatedly. Both players discount the future at the same rate $\delta \in (0, 1]$. At every period, the seller can produce at zero cost a non-storable object that is put up for sale to the buyer.[7] This buyer has valuation θ_i for the good, where $\theta_i \in \Theta = \{\theta_L, \theta_H\}$. Let call θ_L (θ_H) the low-type buyer (high-type buyer), which sometimes is denoted by the subscript L (H). Valuation remains constant over time and is the buyer's private information. The probability of a high-type buyer is denoted by p_H, and of a low-type buyer by $p_L = 1 - p_H$. This p_H indicates the prior of the seller.

A mechanism Γ_r in period r specifies a message set M_r and a decision function $y_r = (x_r, w_r)$, where $x_r : M_r \to [0, 1]$ is the allocation rule and $w_r : M_r \to \mathbb{R}$ is the payment rule. Then, each element $m_r \in M_r$ commits the seller to implement the allocation rule $x_r(m_r)$ and requires for the buyer the payment $w_r(m_r)$.

The seller has imperfect commitment. This is, during the first period the seller can commit herself to a mechanism for the current period but not to a mechanism for the next period. So, at the beginning of the first period the seller chooses a mechanism $\Gamma_2 \in \Upsilon$ given her prior about facing a high-type, where Υ is the space of mechanisms. Next, the buyer observes the mechanism. His strategy specifies the probability $q_i(m_2)$ with which the buyer sends each message, where $q_i : M_2 \to [0, 1]$, for $i \in \{L, H\}$ and that verifies $\sum_{m_2 \in M_2} q_i(m_2) = 1$. The buyer can always choose not to participate in the proposed mechanism.[8] In this case, he gets zero instant payoffs but he can choose to participate in the second period. Next, the seller observes the message sent by the buyer, implements the mechanism and updates her belief about facing a high-type buyer. This new belief, $p_{H,2}(m_2)$, is updated following the mapping $p_{H,2} : M_2 \to [0, 1]$. In the following, $p_{L,2}(m_2)$ indicates $1 - p_{H,2}(m_2)$ and $p_2(m_2)$ indicates the vector of posteriors $(p_{L,2}(m_2), p_{H,2}(m_2))$. Updated beliefs constitute the state variable for the next period. Then, at the beginning of period $r = 1$ the seller chooses a new mechanism $\Gamma_1 \in \Upsilon$ given her updated beliefs, the buyer observes Γ_1 and chooses his strategy in response. The seller observes the new message, implements the new mechanism and the game finishes.

[6] Notation is as similar as possible to theirs in order to simplify the reference to their work.

[7] All results fo this paper hold for any constant production cost strictly less than the minimum possible value that the buyer is willing to pay.

[8] Note that the definition of the mechanism requires participation. The model takes the usual convention that the buyer can decide whether to participate or not, getting zero payoffs in the last case. This convention is discussed later, with the individual rationality constraint (IR). Alternatively, it is possible to include a message in M_2 that represents no participation.

2.1 Equilibrium concept

At every period, given M_r, the seller looks for the Perfect Bayesian Equilibrium (PBE) of this game. Hence, choosing the appropriate Γ_r, the seller can choose a particular PBE.

Denote by $v_r(m_r)$ and $u_{i,r}(m_r)$ to the seller's and buyer's *instant* payoff, respectively, when the buyer with valuation θ_i sends the message m_r, i.e.

$$v_r(m_r) = w_r(m_r),$$
$$u_{i,r}(m_r) = x_r(m_r)\theta_i - w_r(m_r).$$

Let $V_1 : [0,1]^2 \to \mathbb{R}$ and $U_{i,1} : [0,1]^2 \to \mathbb{R}$ represent the continuation values for each player when $r = 2$.[9]

Consequently, given a prior $p \equiv (p_L, p_H)$, the seller's problem at period $r = 2$ is to choose (q_2, p_2, Γ_2) that maximize:[10]

$$\sum_{i\in\Theta}\sum_{m_2\in M_2} p_i q_i\left(m_2\right)\left(v_2(m_2) + \delta V_1\left(p_2(m_2)\right)\right), \tag{1}$$

where $q_2 \equiv \left(q_2\left(m_2\right)\right)_{m_2\in M_2}$, $q_2\left(m_2\right)$ indicates the vector $(q_L(m_2), q_H(m_2))$, and $p_2 \equiv \left(p_2\left(m_2\right)\right)_{m_2\in M_2}$. Notice that when the buyer is indifferent between different messages, he randomizes between them. The seller knows this but she does not observe which probability the buyer chooses for each message. Assume that she can always select the best equilibrium between all the possible ones as is usual in mechanism design.

Seller's objective at (1) is subject to the following constraints:

- The buyer's Incentive Compatibility ($IC_{i,2}$): the buyer chooses his optimal reporting strategy, i.e.,

$$\sum_{m_2\in M_2} q_i\left(m_2\right)\left(u_{i,2}(m_2) + \delta U_{i,1}\left(p_2\left(m_2\right)\right)\right) \geq \tag{2}$$
$$\sum_{m_2\in M_2} q_i'\left(m_2\right)\left(u_{i,2}(m_2) + \delta U_{i,1}\left(p_2\left(m_2\right)\right)\right)$$

  for $i \in \{L, H\}$, and for all $q_i'\left(m_2\right)$.

- The buyer's Individual Rationality ($IR_{i,2}$): The buyer's individual rationality constraint has to be satisfied:

$$p_i\left[\sum_{m_2\in M_2} q_i\left(m_2\right)\left(u_{i,2}(m_2) + \delta U_{i,1}\left(p_2\left(m_2\right)\right)\right) - \delta\bar{U}_{i,1}\right] \geq 0 \tag{3}$$

  for $i \in \{L, H\}$, where $\bar{U}_{i,1}$ is the continuation value when the buyer choose not to participate in the mechanism Γ_2. Let assume $\bar{U}_{i,1} = 0$ since, as it will be shown later, this is the case at the optimal contract (since it is possible to assume any belief for the out-of-equilibrium message).

[9] Continuation values depends on the vector of priors at the beginning of the period. Since there are two types, the vector of priors is completely determined by the prior about facing a high type, i.e. $p_{H,r+1}$. Then, later in the paper, and with some abuse of notation, continuation values will be represented as depending only in that prior.

[10] In Bester and Strausz speficifaction, it is allowed $v_{i,2}(m_2) \neq v_{j,2}(m_2)$ when $i \neq j$ giving $\sum_{i\in\Theta}\sum_{m_2\in M_2} p_{i,3}q_{i,2}\left(m_2\right)\left(v_{i,2}(m_2) + \delta V_{i,1}(p_2)\right)$.

- And finally, for each message, the seller's updated belief $p_{i,2}(m_2)$ has to be consistent with Bayes' rule (BR_2) whenever possible:

$$p_{i,2}(m_2)\sum_{j\in\Theta} p_j q_j(m_2) = p_i q_i(m_2). \tag{4}$$

It follows that the seller's problem with imperfect commitment at $r=2$ is given by:

$$V_2(p_3) = \underset{\{q_2,p_2,\Gamma_2\}}{Max} \sum_{i\in\Theta}\sum_{m_2\in M_2} p_i q_i(m_2)\left(v_2(m_2)+\delta V_1\left(p_2(m_2)\right)\right), \tag{5}$$
subject to $(2)-(4)$.

The outcome (q_2, p_2, Γ_2) is *incentive feasible* if it satisfies (2)-(4) for all $\theta_i \in \Theta$. Additionally, it is *incentive efficient* if it satisfies (5), the seller chooses the best outcome among all of the incentive feasible ones. An *optimal mechanism* is a mechanism Γ_2 that belongs to an incentive efficient outcome (q_2, p_2, Γ_2). Finally, (q_2, p_2, Γ_2) and (q_2', p_2', Γ_2') are *payoff equivalent* if they leave the seller and the buyer (of every possible type) with the same expected payoffs, i.e.

$$\sum_{i\in\Theta}\sum_{m_2\in M_2} p_i q_i(m_2)\left(v_2(m_2)+\delta V_1\left(p_2(m_2)\right)\right) =$$
$$\sum_{i\in\Theta}\sum_{m_2'\in M_2'} p_i q_i\left(m_2'\right)\left(v_2(m_2')+\delta V_1\left(p_2'(m_2)\right)\right),$$
$$\sum_{m_2\in M_2} q_i(m_2)\left(u_{i,2}(m_2)+\delta U_{i,1}\left(p_2(m_2)\right)\right) =$$
$$\sum_{m_2'\in M_2'} q_i'\left(m_2'\right)\left(u_{i,2}(m_2')+\delta U_{i,1}\left(p_2'(m_2)\right)\right), \quad i\in\{L,H\}.$$

2.2 Revelation Principle

This subsection shows that, i) we can restrict to direct mechanisms, ii) p_2 is always determined by Bayes' rule (consequently there are not out-of-equilibrium beliefs) and, iii) it is enough to consider a subset of all possible q_2.

A direct mechanism is a mechanism in which the message set is the buyer's type set, i.e., $M_r = \Theta$. In this case, the buyer's strategy is to report each type with some probability, i.e., $q_i : \Theta \to [0,1]$, with $\sum_{m_r\in\Theta} q_i(m_r) = 1$. Bester and Strausz (2001) provides a revelation principle for environments with imperfect commitment, including the multistage contracting case as the problem proposed in this work. Based on this revelation principle, the solution of (5) using direct mechanisms, i.e.:

Lemma 1 *Assume a state p_{r+1}. Any solution (q_r, p_r, Γ_r) for (5) is payoffs equivalent to a solution $\left(\hat{q}_r, \hat{p}_r, \hat{\Gamma}_r\right)$ where $\hat{\Gamma}_r$ is a direct mechanism and where the buyer reports his type with positive probability, i.e., $\hat{q}_i(i) > 0\ \forall\ i\in\{L,H\}$.*

Proof. This lemma is a direct application of Proposition 2 and its corollary at Bester and Strausz (2001). ■

Bester and Strausz (2001) shows that it is sufficient for the mechanism designer to consider mechanisms in which the set of messages has equal cardinality to the type space. Moreover, they show that we can associate each message with a type that plays the message with positive probability.

A consequence is that the mechanism designer can be restricted to outcomes (q_r, p_r, Γ_r) where the mechanism has Θ as the message set. Then, as every message that belongs to Θ is reported with positive probability, she can always associate a message with the corresponding type. That is, she asks every type to report the truth with positive probability, i.e., $q_H > 0$ and $1 - q_L > 0$ where q_H (q_L) is the probability that a high-type buyer (low-type buyer) sends a high-type message.

Notice that this revelation principle differs from the standard one (see Myerson 1981) in that there is no guarantee that the buyer reports his true type with certainty. Even so, truthful reporting is always an optimal strategy for the buyer and he still plays it with positive probability.

Given some mechanism Γ_r, (2) requires that any message which is played with positive probability must be optimal for the buyer. From the revelation principle either $q_H = 1$ ($q_L = 0$), in which case (2) requires that the high-type (low-type) prefers to report the truth, or $q_H < 1$ ($q_L > 0$) in which case (2) requires indifference between both messages. Hence, in this two-period setting, (2) can be simplified to:

$$
\begin{aligned}
IC_{H,2}: &\quad u_{H,2}(h) + \delta U_{H,1}(p_2(h)) \geq u_{H,2}(l) + \delta U_{H,1}(p_2(l)) \;\; \text{with equality if } 1 - q_H > 0, \\
IC_{L,2}: &\quad u_{L,2}(l) + \delta U_{L,1}(p_2(l)) \geq u_{L,2}(h) + \delta U_{L,1}(p_r(h)) \;\; \text{with equality if } q_L > 0,
\end{aligned}
$$

where, from now on, h and l indicates a high-type and low-type message, respectively.

As every message is sent with positive probability by at least one type, (4) is always satisfied. As a consequence, the posteriors are completely determined by Bayes' rule and p_2 is a redundant variable of optimization.

Without loss of generality, the analysis is concentrated on those incentive feasible outcomes such that $q_H \geq q_L$. For those incentive feasible outcomes such that $q_L > q_H$, simply define a new mechanism in which the role of each message is interchanged (the corresponding Lemma and its proof are relegated to the Appendix). Notice that $q_H \geq q_L$ implies $p_{H,2}(h) \geq p_H \geq p_{H,2}(l)$ by Bayes' rule.

3 Optimal Selling Mechanism

This section solves the two-period case of the previous problem using backward induction. It is proved that the optimal selling mechanism in both periods can be implemented by price posting, i.e., a take-it-or-leave-it offer.

Definition 1 *A Price Posting Mechanisms in period r -in what follows price posting- is an indirect mechanism with two possible messages "take − it" or "leave − it", with allocation and payment rules according to:*

$$
x_r(m_r) = \begin{cases} 1 & if \quad m_r = take - it, \\ 0 & if \quad m_r = leave - it. \end{cases} \; ; \quad w_r(m_r) = \begin{cases} z_r & if \quad m_r = take - it, \\ 0 & if \quad m_r = leave - it. \end{cases}
$$

where $z_r \in \Re$ is the price asked by the seller.

3.1 Period r=1

Last period solution is very well known. To get it, instead of using Bester and Strausz, it is possible to apply standard mechanism design (see for example Bolton and Dewatripont, 2005) without loss of generality.

When seller's belief in facing a high-type buyer is larger than θ_L/θ_H, she proposes a separation mechanism: high-type buyer receives the good with certainty and pays his valuation, while a low-type buyer does not receive the good and pays zero. On the other hand, when that belief is lower θ_L/θ_H, she proposes pooling: every buyer gets the good and payment is the low-type valuation. A belief equal to θ_L/θ_H is the limit between both mechanisms, i.e., the seller is indifferent between both mechanisms. She can even propose any mechanism in which allocation for message l is any value between 0 and 1. However, the seller cannot do better than in the pooling or separation cases. From now on, and to simplify the notation, allocation for message l in this case is considered equal to zero.[11]

Remark 1 *The optimal mechanism at $r = 1$ can be implemented by the following price posting:*
a) if $p_{H,2} \geq \theta_L/\theta_H$, $w_1(take-it) = \theta_H$,
b) if $p_{H,2} < \theta_L/\theta_H$, $w_1(take-it) = \theta_L$.

Proof. The proof consists in showing that there is an indirect mechanism with the properties of a take-it-or-leave-it offer that is payoff equivalent to our optimal direct mechanism. Because this is a one period game, it is straightforward. See the Appendix for the details. ■

3.2 Period r=2

Hence continuation values for the high-type buyer, for the low-type buyer and for the seller, are respectively:

$$\begin{aligned}
&U_{H,1}(p_{H,2}) = \Delta\theta \mathbf{I}_{[0,\theta_L/\theta_H)}(p_{H,2}), \\
&U_{L,1}(p_{H,2}) = 0, \quad \forall\, p_{H,2}, \\
&V_1(p_{H,2}) = p_{H,2}\theta_H \left[1 - \frac{U_{H,1}(p_{H,2})}{\Delta\theta}\right] + \frac{U_{H,1}(p_{H,2})}{\Delta\theta}\theta_L.
\end{aligned} \tag{6}$$

As the continuation values for the low-type are zero for every prior, his payoffs at $r = 2$ are only his instant payoff, while the payoffs for the high-type buyer are the sum of the instant payoffs and his continuation value at (6).

The following lemma states proves that seller's problem at (5) for $r = 2$, after simplifications of Section 2.1, is equivalent to (7).

[11] Choosing any other value of allocation for message l in that case does not change main results of the paper. Bester and Strausz specification allows the possibility of giving to the seller the option of choosing this value at first period. Using an example, it can be shown that at first period the seller prefers $x_1(l) = 1$ when her prior is lower than θ_L/θ_H, and $x_1(l) = 0$ when her prior is higher than θ_L/θ_H. Including this action for the seller complicates the model without upsetting the results.

Lemma 2 *The seller's problem at (5) for $r = 2$ is equivalent to the reduced program:*

$$\begin{aligned}
&\underset{\{q_2,\Gamma_2\}}{Max} \sum_{i=L,H} \sum_{m_2=l,h} p_i q_i(m_2) \left[v_2(m_2) + \delta V_1(p_{H,2}(m_2))\right], \quad \textit{subject to,} \qquad (7)\\
&IC^*_{H,2}: \quad u_{H,2}(h) + \delta U_{H,1}\left(p_2\left(h\right)\right) = u_{H,2}(l) + \delta U_{H,1}\left(p_2\left(l\right)\right),\\
&IR^*_{L,2}: \quad u_{L,2}(l) + \delta U_{L,1}\left(p_2\left(l\right)\right) = 0,\\
&SMC_2: x_2(h) - x_2(l) \geq \delta\frac{U_{H,1}\left(p_2(l)\right)}{\Delta\theta} - \delta\frac{U_{H,1}\left(p_2(h)\right)}{\Delta\theta}, \quad \textit{with equality if } q_L > 0,\\
&BR_2: \quad p_{i,2}\left(m_2\right) = \frac{p_i q_i\left(m_2\right)}{\sum\limits_{k=L,H} p_k q_k(m_2)}, \; m_2 = l, h, \; i = L, H\\
&x_2 \in [0,1], \; q_H > 0, \; q_L < 1.
\end{aligned}$$

Proof. See the Appendix ■

In this reduced program, the seller considers a binding incentive compatibility constraint for the high-type ($IC^*_{H,2}$), a binding individual rationality for the low-type ($IR^*_{L,2}$) and a new constraint, the *Sequential Monotonicity Condition for* $r = 2$ (SMC_2), which replaces the incentive compatibility of low-type.

The proof has three steps. First, the incentive compatibility of the high-type ($IC_{H,2}$) jointly with the individual rationality of the low-type ($IR_{L,2}$) imply that individual rationality of the high-type ($IR_{H,2}$) is always satisfied. Second, the seller can increase the payment for both messages the same amount without changing continuation values and while keeping satisfied both incentive compatibility constraints and the individual rationality of the low-type buyer. It is optimal for the seller to make this increment in payments until the one for low-type message extracts all his surplus, resulting in $IR^*_{L,2}$. This payment is the maximum value that the low-type buyer can pay without retreating from the mechanism. Suppose that high-type continuation values are fixed. Once the seller fixes previous payment, she continues increasing the high-type message's payment until the high-type buyer is indifferent to reporting the truth or not in such a way that continuation values do not change, i.e., $IC^*_{H,2}$ for those continuation values. For different fixed continuation values, the seller could follow the same procedure getting again $IC^*_{H,2}$ for the new continuation values. Hence, the outcome that maximizes seller's payoff must be one in which $IC_{H,2}$ is binding. Note that, because the high-type buyer is indifferent to both messages, the requirement that he must tell the truth with positive probability is satisfied. Finally, assuming $IC^*_{H,2}$ and $IR^*_{L,2}$, the incentive compatibility of the low-type buyer ($IC_{L,2}$) is equivalent to the SMC_2. This new restriction plays a similar role as the monotonicity condition of the static case, which asks to allocation to be increasing in the buyer type. In fact, notice that if $\delta = 0$, this model would collapse to the static case and the SMC_2 to the monotonicity condition. In this dynamic framework, the SMC_2 is more restrictive than the monotonicity condition.[12] It still asks that the current allocation increases in the buyer type. It also requires that the difference in current allocations must be at least as large as the difference between the discounted continuation values (weighted by $\Delta\theta$) that the high-type buyer gets by lying and by telling the truth.

Operating with $IC^*_{H,2}$ and $IR^*_{L,2}$ and plugging them into the seller's objective function, the seller's

[12] The case under which the SMC_2 is weaker than the standard monotonicity condition can be ruled out since $p_2(h) \geq p_2(l)$ and hence $U_{H,1}\left(p_2(l)\right) \geq U_{H,1}\left(p_2(h)\right)$.

problem at (7) becomes

$$\begin{aligned} & \underset{\{q_2,x_2\}}{Max}\ x_2(l)\theta_L + \rho_{H,3}\left(x_2(h) - x_2(l)\right)\theta_H + \delta\rho_{H,3}\left[U_{H,1}\left(p_{H,2}(h)\right) - U_{H,1}\left(p_{H,2}(l)\right)\right] \\ & \qquad + \delta\left[\rho_{H,3}V_1(p_{H,2}(h)) + \left(1-\rho_{H,3}\right)V_1(p_{H,2}(l))\right], \\ & \text{subject to,} \\ & SMC_2,\ BR_2,\ x_2 \in [0,1],\ q_H > 0,\ \text{and}\ q_L < 1. \end{aligned} \tag{8}$$

where $\rho_{H,3} = (p_H q_H + p_L q_L)$ is the total probability of observing a message h.

To solve the game it is useful to introduce the following definition.

Definition 2 *A mechanism induces significant* learning *when* $U_{H,1}\left(p_{H,2}(l)\right) - U_{H,1}\left(p_{H,2}(h)\right) \neq 0$.

Suppose that there is no difference between buyer's continuation values when he lies and when he tells the truth. From (6), seller's posteriors for both messages are either above θ_L/θ_H or either below it. Therefore, no matter the message sent by the buyer in the first period, in the former case the seller is proposing at $r = 1$ a price posting equal to θ_H and in the latter case, a price posting equal to θ_L. Note that seller's expected continuation value is linear on p_H and equal to $V_1(p_H)$.[13]

Suppose on the other hand, that there is difference between buyer's continuation values when he lies and when he tells the truth. From (6) and since $q_H \geq q_L$, it must be that the seller's posterior when she observes a message h is higher or equal to θ_L/θ_H and when she observes a messages l is lower than it. Hence, the difference between buyer's continuation values is equal to $\Delta\theta$. Therefore, the seller is proposing at $r = 1$ a different price posting depending on the message observed in the first period: θ_H in case of observing message h and θ_L in case of l.

The interpretation indicates that learning becomes relevant when it induces the seller to offer a different mechanism in the next period.

The problem is splitted into two subproblems to solve it. First, taking q_2 as given, it is solved with respect to x_2. Second, it is solved with respect to q_2 using the allocations obtained in the first step.[14]

Consider the first subproblem. Note that because the seller's payoffs are increasing in $x_2(h)$ and that an increment of $x_2(h)$ relaxes the SMC_2, then the optimal $x_2(h)$ is 1. Two different situations arise to obtain the allocation for message l: when low-type buyer reports his type with probability one (i.e., $q_L = 0$) and when he lies with positive probability (i.e., $q_L \neq 0$).

In the first case, the allocation when a message l is sent depends on the value of $\rho_{H,3}$ and is given by

$$x_2(l) = \begin{cases} 0 & if \quad \rho_{H,3} > \frac{\theta_L}{\theta_H}, \\ \alpha_2 & if \quad \rho_{H,3} = \frac{\theta_L}{\theta_H}, \\ \mu & if \quad \rho_{H,3} < \frac{\theta_L}{\theta_H}, \end{cases}, \tag{9}$$

with $\alpha_2 \in [0, \mu]$ and $\mu = \min\left\{1, 1 - \delta\frac{U_{H,1}\left(p_{H,2}(l)\right)}{\Delta\theta} + \delta\frac{U_{H,1}\left(p_{H,2}(h)\right)}{\Delta\theta}\right\}$. When $\rho_{H,3} = \theta_L/\theta_H$, the seller's payoffs are constant for any $x_2(l) \in [0, \mu]$. Then, to simplify the analysis assume $x_2(l) = 0$

[13] In other words, the convex combination of seller's continuation values, $\rho_{H,3}V_1(p_{H,2}(h)) + \left(1-\rho_{H,3}\right)V_1(p_{H,2}(l))$ is equal to $V_1(p_H)$. There are two possible situations when there is no significant learning: when $p_H < \frac{\theta_L}{\theta_H}$ and seller's continuation values are $V_1(p_{H,2}(h)) = V_1(p_{H,2}(l)) = \theta_L$ (i.e., $V_1(p_H) = \theta_L$); when $p_H \geq \frac{\theta_L}{\theta_H}$ and seller's continuation values are $V_1(p_{H,2}(h)) = p_{H,2}(h)\theta_H$ and $V_1(p_{H,2}(l)) = p_{H,2}(l)\theta_H$ (i.e., $V_1(p_H) = p_H\theta_H$).

[14] It is used the general property $\underset{\{x,y\}}{Max} f(x,y) = \underset{\{x\}}{Max}\left(\underset{\{y\}}{Max} f(x,y)\right)$.

at $\rho_{H,3} = \theta_L/\theta_H$.

When $q_L \neq 0$, the low-type is indifferent to both messages and the SMC_2 holds with equality, restricting the value of $x_2(l)$, which is now given by

$$x_2(l) = 1 - \delta \frac{U_{H,1}(p_{H,2}(l))}{\Delta\theta} + \delta \frac{U_{H,1}(p_{H,2}(h))}{\Delta\theta}. \tag{10}$$

To solve the second subproblem, differentiate those cases where the good is allocated with zero probability in case of message l ($x_2(l) = 0$) and with positive probability ($x_2(l) \neq 0$).

Definition 3 *A mechanism has* SMC non-binding *if* $x_2(l) = 0$ *and* SMC binding *if* $x_2(l) \neq 0$.

In both cases, there exist incentive feasible mechanisms that induces significant *learning* and with *no-learning* (i.e., when learning is not relevant). Notice that allocation $x_2(l) = 0$ occurs only when low-type buyer reports his type with probability one and, from (9), when $\rho_{H,3} \geq \theta_L/\theta_H$. On the other hand, $x_2(l) \neq 0$ occurs either when $q_L = 0$ and $\rho_{H,3} < \theta_L/\theta_H$, or when $q_L \neq 0$. In both cases, by (9) or (10), respectively, $x_2(l) = 1 - \delta$ when there is *learning* and $x_2(l) = 1$ when there is *no-learning.*

Hence there are four candidates to be the optimal mechanisms depending on the prior (detailed discussion is relegated to the Appendix). These four candidates are enumerated next, including the maximum payoff that the seller makes in each case.

1. *SMC binding with no-learning* (SMC*+NL): with $x_2(h) = 1$, $x_2(l) = 1$, and $q_H = q_L \neq 0$,

$$\theta_L + \delta \max\{p_H\theta_H, \theta_L\}.$$

2. *SMC binding with learning* (SMC*+L): with $x_2(h) = 1$, $x_2(l) = 1 - \delta$, and $q_H = 1$, for any q_L,

$$\theta_L + \delta p_H \theta_H.$$

3. *SMC non-binding with learning* (SMC+L): (defined for $p_H \geq \frac{\theta_L}{\theta_H}$) with $x_2(h) = 1$, $x_2(l) = 0$, and $q_H = 1$, $q_L = 0$,

$$p_H\theta_H + \delta\theta_L.$$

4. *SMC non-binding with no-learning* (SMC+NL): (defined for $p_H \geq p^*$, where $p^* = \frac{\theta_L}{\theta_H^2}\Delta\theta + \frac{\theta_L}{\theta_H}$) with $x_2(h) = 1$, $x_2(l) = 0$, and $q_H = \frac{p_H\theta_H - \theta_L}{p_H\Delta\theta}$, $q_L = 0$,

$$\frac{p_H\theta_H - \theta_L}{\Delta\theta}\theta_H + \delta p_H\theta_H.$$

Hence, the first candidate is a pooling mechanism in which both types randomizes between messages. The second one is a semi-separation mechanism under which the high-type reports his type with certainty while the low-type randomizes. The third candidate is a separation mechanism under which both types reports the truth with probability one. Finally, SMC+NL is another semi-separation mechanism but at different terms than previous one in which the high-type is randomizing between messages and the low-type reports his type with probability one. It is proved later that

SMC*+L is not payoff equivalent to a price posting (in Proposition 2), while the remaining three mechanisms are (in Corollary 1).[15] For the moment, assume this is true.

Notice that at a prior $p_H = \frac{\theta_L}{\theta_H}\left(\frac{\theta_H+\delta\Delta\theta}{\theta_L+\delta\Delta\theta}\right)$, the seller is indifferent between a separation price posting (SMC+L) and a semi-separation one (SMC+NL). Denote this prior as τ_2 and assume, without a loss of generality, that when $p_H = \tau_2$ the seller proposes the semi-separation price posting. At a prior equal θ_L/θ_H, pooling, non-price posting and separation price posting give the same payoff to the seller. Denote it with τ_2^* and, again without a loss of generality, assume that the seller proposes a separation price posting at that prior.

It is straightforward to check under which prior the seller finds optimal to choose each mechanism. This is stated in next proposition.

Proposition 1 *The optimal selling mechanism verifies that:*

- *if* $p_H < \tau_2^*$, (SMC binding with no-learning) *with* $x_2(h) = 1$, $x_2(l) = 1$, $w_2(h) = \theta_L$, $w_2(l) = \theta_L$, *and* $q_H = q_L \neq 0$.

- *if* $p_H \in [\tau_2^*, \tau_2)$, (SMC non-binding with learning) *with* $x_2(h) = 1$, $x_2(l) = 0$, $w_2(h) = \theta_H - \delta\Delta\theta$, $w_2(l) = 0$, $q_H = 1$ *and* $q_L = 0$.

- *if* $p_H \geq \tau_2$, (SMC non-binding with no-learning) *with* $x_2(h) = 1$, $x_2(l) = 0$, $w_2(h) = \theta_H$, $w_2(l) = 0$, $q_H = \frac{p_H\theta_H - \theta_L}{p_H\Delta\theta}$ *and* $q_L = 0$.

Proof. Directly from previous cases. It only remains to get $w_2(h)$ and $w_2(l)$. These variables are just mechanic substitutions in $IC_{H,2}^*$ and $IR_{L,2}^*$. ■

The thought behind the results of the proposition are the following. The first case corresponds with the seller being *pessimistic* ($p_H < \tau_2^*$). In this case she proposes pooling, i.e., she sells at a price equal to the low-type value while resigns from learning. The seller has only one alternative: to propose the non-price posting SMC*+L in which she sells to both types at different terms and learns after a message h. In this alternative, she would like that payments for each message were different but close to each other. However, the SMC_2 restricts seller's capability of doing that, making this mechanism dominated by pooling.

When the seller is *optimistic* ($p_H \geq \tau_2^*$), a mechanism SMC+L or a SMC+NL is the optimal one. In both cases, when observing a message h, the optimal mechanism in next period is to sell only to a high-type buyer at a price equal to his value. In particular, if $p_H \geq \tau_2$ (the seller is *extremely optimistic*), she prefers a semi-separation price posting. In such a case, there is *no-learning* and the optimal mechanism is such that she sells only to the high-type buyer in the second period no matter the message observed in the first period.[16] Therefore, in the second period, the buyer always makes zero surplus. On the other hand, when the seller is *moderately optimistic, i.e.,* $p_H \in \left[\frac{\theta_L}{\theta_H}, \tau_2\right)$, the optimal mechanism induces *learning*. In next period, she sells to a high-type buyer at a price equal to his value only in the case of observing h in the first period. Otherwise, she sells to both types at a low-type value price. In this mechanism, the seller is paying a bribe to incentive the high-type buyer to reveal his type. This bribe is equal to his discounted future losses by being discriminated

[15] Although it could be thought that it is enough to check whether allocations in these direct mechanisms are 1 or 0 (as in Definition 1), it is also necessary to take into account continuation values. The reason is that it is possible that a direct mechanism with allocations different to 1 and 0, and with some particular continuation values, is payoff equivalent to a price posting with different continuation values.

[16] The seller picks a q_H that "commits" her to sell in the second period to the high-type buyer at a price equal to his valuation while also asking a high payment in the first period. This q_H is lower than one (assigning positive probability of lying to a high-type), keeping her optimistic enough in the case of observing a message l.

in the second period. Therefore, the buyer makes zero surplus in the second period in case the of reporting h or positive surplus in case of reporting l.[17] Alternatively, using a non-price posting, the seller could obtain the same posteriors (and as consequence the same continuation values) but, since she has to keep both types indifferent between messages, she has to ask for a lower payment in the first period.

Next figure summarizes optimal belief dynamic for each prior.

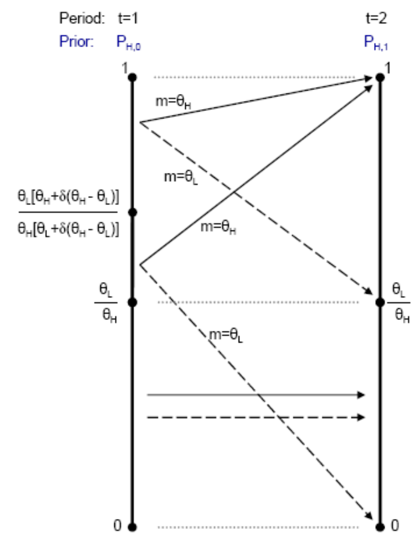

Figure 1: Optimal belief dynamic under different priors. Vertical line at the left represents the prior at r=2. Vertical line at the right represents the posteriors. Full-line arrow is the belief dynamic under a message h. Dot-line arrow is the belief dynamic under a message l.

Proposition 1 gives a characterization of the optimal selling mechanisms for any prior. It was assumed above that they were payoff equivalent to price posting. The following corollary states that this assumption was right. The proof proposes an alternative outcome $(\hat{q}_2, \hat{p}_2, \hat{\Gamma}_2)$ where $\hat{\Gamma}_2$ is a price posting mechanism (i.e., an indirect mechanism). Next, its check (for any prior) whether this outcome is payoff equivalent to the incentive efficient outcome (q_2, p_2, Γ_2) that solves (7) and which contains the optimal selling mechanism characterized in previous propositions (which are direct mechanisms).

Corollary 1 *The optimal selling mechanism at $r = 2$ can be implemented by a price posting. In particular:*
1) when $p_H < \frac{\theta_L}{\theta_H}$, the price is θ_L, both types always buy,
2) when $p_H \in \left[\frac{\theta_L}{\theta_H}, \tau_2\right)$, the price is $\theta_H - \delta\Delta\theta$, the high-type buyer always buys and low-type buyer never buys, and
3) when $p_H \geq \tau_2$, the price is θ_H, the high-type buyer randomizes with probability $\frac{p_H\theta_H - \theta_L}{p_H\Delta\theta}$ and low-type buyer never buys.

Proof. See the Appendix ■

These results reassert that the price posting mechanisms are the optimal mechanisms when the seller is not restricted to take-it-or-leave-it offers.

4 Difference in patience

This section shows that when buyer and seller differ in their patience price posting is no longer optimal.

[17] To offer this bribe, the seller considers that the high-type is going to report the truth with probability one, i.e., she picks $q_H = 1$.

Suppose that both players have different discount factors: β for the seller, δ for the buyer. Previous section analyzed $\beta = \delta$, showing that price posting was optimal. When $\beta \neq \delta$, seller's problem is similar to (8) changing only the discount factor that affects seller's continuation values (i.e., δ is replaced by β).

Using the same procedure than in Section 3, the seller gets the following maximum payoffs in each case:

1. *SMC*+NL:* with $x_2(h) = 1$, $x_2(l) = 1$, $w_2(h) = \theta_L$, $w_2(l) = \theta_L$, and $q_H = q_L \neq 0$.

$$\theta_L + \beta \max\{p_H \theta_H, \theta_L\}.$$

2. *SMC*+L:* with $x_2(h) = 1$, $x_2(l) = 1 - \delta$, $w_2(h) = \theta_L$, $w_2(l) = (1-\delta)\theta_L$, and $q_H = 1$ (for any $\beta \neq \delta$), $q_L = 0$ (if $\beta > \delta$), $q_L \to 1$ (if $\beta < \delta$),

$$\begin{aligned} &\theta_L + (\beta - \delta)(1 - p_H)\theta_L + \beta p_H \theta_H, \quad \text{if } \beta > \delta, \\ \rightarrow \quad &\theta_L + \beta p_H \theta_H, \quad \text{if } \beta < \delta. \end{aligned}$$

3. *SMC+L:* (defined for $p_H \geq \frac{\theta_L}{\theta_H}$) with $x_2(h) = 1$, $x_2(l) = 0$, $w_2(h) = \theta_H - \delta\Delta\theta$, $w_2(l) = 0$, and $q_H = 1$, $q_L = 0$,

$$p_H \theta_H + (\beta - \delta) p_H \Delta\theta + \beta\theta_L.$$

4. *SMC+NL:* (defined for $p_H \geq p^*$) with $x_2(h) = 1$, $x_2(l) = 0$, $w_2(h) = \theta_H$, $w_2(l) = 0$, and $q_H = \frac{p_H\theta_H - \theta_L}{p_H \Delta\theta}$, $q_L = 0$,

$$\frac{p_H\theta_H - \theta_L}{\Delta\theta}\theta_H + \beta p_H \theta_H.$$

Suppose the seller is more patient than the buyer ($\beta > \delta$). Now, in the SMC*+L mechanism, she finds optimal to consider that both types are reporting their types with certainty. Therefore, this mechanism is working as a separation mechanism. It turns out that when the seller is more patient and when she is pessimistic, SMC*+L may give larger payoff to the seller than pooling. Next proposition states this result. It additionally proves that SMC*+L cannot be implemented by a price posting.

Proposition 2 *For any $\beta > \delta$, there exists a prior $p_H < \frac{\theta_L}{\theta_H}$ such that price posting is not optimal.*

Proof. See the Appendix. ■

The intuition behind the preposition can be understood with the following trick. Suppose a seller who is very patient faces a buyer who is not (with $\delta \to 0$). Suppose also that the seller is pessimistic about facing a high-type buyer. From previous analysis, the seller can propose either pooling or a non-price posting mechanism. In the former case, she resigns to learning by selling in both periods with probability one. Under the latter, she sells to both types at different terms, learning when she observes a message h. Since the buyer does not value the future, he behaves as he was in the one period game, reporting the truth with certainty. Additionally, the seller's cost of inducing separation with this mechanism is proportional to δ. Hence, the non-price posting improves her payoff with respect to pooling. In case that the seller is optimistic, again with a separation or semi-separation price posting, she can learn as much as under the non-price posting mechanism while getting larger instant expected payoff.

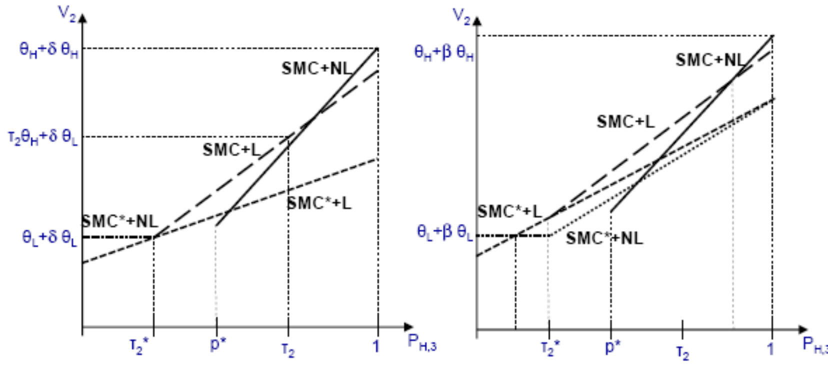

Figure 2: Seller's payoffs under different mechanisms when $\delta = \beta$ (at the left) and whne $\delta < \beta$ (at the right). In Dot-line case **SMC*+NL**; in Small-dash-line case **SMC*+L** (coincident with case **SMC*+NL** when $p_{H,3} \geq \tau_2^*$ in the graph at the left); in Dash-line case **SMC+L;** in Solid-line case **SMC+NL**.

The following example illustrates, for a particular prior, how large has to be the difference between discount factors to have that price posting is not optimal.

Example 1 Different Discount Factors:

Let consider $\theta_L = 1$, $\theta_H = 2$, *a prior* $p_H = \frac{1}{3}$ *and* $\beta = 1$. *Case SMC+L and SMC+NL are not defined for* $p_H < \frac{\theta_L}{\theta_H}$.

Seller's payoffs using a mechanism from case SMC+NL are equal to* $\theta_L + \beta\theta_L$, *i.e.* $V_2 = 2.0$. *Seller's payoffs using a mechanism from case SMC*+L are equal to* $\theta_L + (\beta - \delta)(1 - p_H)\theta_L + \beta p_H \theta_H$, *i.e.* $V_2 = 1 + (1 - \delta)\frac{2}{3} + \frac{2}{3}$. *Choosing the appropriate value for* δ, *previous seller's payoffs can be larger than* $\theta_L + \beta\theta_L$.

The next chart shows how seller's payoffs change with δ:

Γ_2 / δ	**Case 1** $x_2(h)=x_2(l)=1$, $w_2(h)=w_2(l)=\theta_L$, $q_H=q_L\neq 0$ $V_2=$	**Case 2** $x_2(h)=1$, $x_2(l)=1-\delta$, $w_2(h)=\theta_L$, $w_2(l)=(1-\delta)\,\theta_L$, $q_H=1$, $q_L=0$ $V_2=$
1.00	2.0	1.67
0.75	2.0	1.83
0.50	2.0	2.0
0.25	2.0	2.17
0.00	2.0	---

When $\delta = 0.25$, *a mechanism with SMC*+L (case 2) maximizes seller's payoffs making* $V_2 = 2.17$. *Buyer's payoffs are equal to* 1 *for the high-type buyer and* 0 *for the low-type buyer. This mechanism cannot be implemented by a price posting as was proved in Proposition 2.*

In the example the seller is moderately pessimistic about facing a high-type buyer. This is, she believes that the probably of facing a high-type buyer is small, but it is still large enough to leave room for finding optimal to learn when, at the same time, she is relatively more patient than the buyer. Notice that seller's payoff under the non-price posting is larger than under pooling when $\frac{\beta\Delta\theta p_H}{\theta_L(1-p_H)} > \delta$. In the example, it occurs when $0.5 > \delta$.

4.1 Implications

Hart and Tirole (1988) proved that, in a two period setting, the seller makes the same payoff when selling a durable good than when renting it. This result relies on the fact that they used price posting as the selling mechanism in both cases (i.e., a selling price and a rental price respectively) and buyers are non-anonymous (the seller can track their behavior as in the current model).

The model proposed in the current paper can be interpreted as a renting model of a durable good in which the seller does not restrict herself to use price posting. Hence, Proposition 2 implies that, when the seller is slightly more patient than the buyer, she prefers renting with a non-price posting than renting or selling with a price posting when she is pessimistic and the opposite when she is optimistic. Hence, the selling and the rental model of a durable good are not equivalent anymore.

Additionally, in Hart and Tirole (1988) model, seller's payoff with commitment are larger or equal than seller's payoff with imperfect commitment.[18] The optimal SMC*+L mechanism proposed in this section can also be implemented under commitment. Hence, price posting may not be the optimal mechanism under commitment neither.

This implications are summarized in next chart where: V^1 and V^2 denote seller's payoffs with imperfect commitment and with commitment respectively when she restricts herself to use a price posting and; V^3 denotes seller's payoff when she exclusively uses a non-price posting mechanism.

		Seller's profits in two-period setting	
		Commitment	**Imperfect-commitment**
Price Posting	**Sale**	V^2	V^1
	Rental	V^2	V^1
Non-price posting		V^3	V^3

$V^2 > V^1 > V^3$ when seller is optimistic; $V^3 > V^2 = V^1$ when she is pessimistic

4.2 Goethe's Mechanism

This subsection shows that the model explains why the mechanism proposed by Goethe (see the Introduction) may be optimal when price posting is not.

To prove so, using previous example, it is constructed a variation of the mechanism à la Goethe and its equilibrium. Next, it is shown that payoffs of this mechanism are arbitrarily close to payoffs of the optimal mechanism in the example.

Example 2 Goethe's Mechanism:

Publisher valuations are $\theta_L = 1$ or $\theta_H = 2$. Goethe has a prior $p_H = \frac{1}{3}$. Discount factors are $\beta = 1$ for Goethe and $\delta = 0.25$ for the publisher.

At last period $r = 1$, Goethe uses the optimal price posting mechanism described at the beginning of Section 3.

At $r = 2$, Goethe proposes to the publisher the following mechanism:

[18] In their paper, they use *non-commitment* to indicate what in this paper was called imperfect commitment.

- Goethe sends to a lawyer a sealed envelope with his reservation price $R \in \Re^+$. Previously, Goethe commits with the publisher to the probability with which he will send each possible value of R.[19]

- At the same time, the publisher sends to the lawyer a sealed envelope with his offer $m \in \Re^+$.

- If $m \geq R$, sale takes place at price R (i.e. $x(m) = 1, w(m) = R$). If $m < R$, the good is not sold (i.e. $x(m) = 0, w(m) = 0$).

An equilibrium for this mechanism is:

- Goethe commits to send a reservation price $R_1 = \theta_L + \varepsilon$ with probability p, and $R_2 = \theta_L$ with probability $(1-p)$, where $p = \frac{\delta\Delta\theta}{\Delta\theta - \varepsilon}$.

- High-type reports $m_1 = \theta_L + \varepsilon$ and low-type reports $m_2 = \theta_L$.

Publisher's payoffs for message m_1 and m_2 ($U_{i,2}(m_1)$ and $U_{i,2}(m_2)$ respectively, where $i \in \{L, H\}$) are:

$$\begin{aligned} U_{H,2}(m_1) &= p(\Delta\theta + \varepsilon) + (1-p)\Delta\theta, \\ U_{H,2}(m_2) &= p\delta\Delta\theta + (1-p)(\Delta\theta + \delta\Delta\theta), \\ U_{L,2}(m_1) &= -p\varepsilon, \\ U_{L,2}(m_2) &= 0. \end{aligned}$$

Notice that $U_{H,2}(m_1) > U_{H,2}(m_2)$ and $U_{L,2}(m_2) > U_{L,2}(m_1)$ when $\varepsilon > 0$. It follows that each type reports his respective message with probability one, revealing their types. High-type buyer gets the poem no matter the reservation price sent by Goethe and low-type buyer gets it only in case of R_2. Hence, Goethe's payoff is

$$V_2 = p_H [pR_1 + (1-p)R_2 + \beta\theta_H] + (1-p_H)[(1-p)R_2 + \beta\theta_L].$$

When $\varepsilon \to 0$, types are almost indifferent between messages with $U_{H,2}(\cdot) \to \Delta\theta$ and $U_{L,2}(\cdot) \to 0$. Goethe makes $V_2 \to 2.17$, almost as the optimal mechanism at Example 1. Therefore, Goethe's Mechanism is optimal in the limit.

5 Concluding Remarks

This paper establishes that the optimal selling mechanism when an uniformed seller with imperfect commitment faces the same consumer in a two periods game is to post a price in each one. This result holds whenever there is no difference in discount factors. Otherwise price posting is not optimal and the Goethe's Mechanism can be rationalized. The method used to solve this problem relies on the procedure proposed by Bester and Strausz (2001).

This paper can be extended in many directions. The more natural extension is to generalize the model for more periods.[20] Allowing more than two periods provides a richer environment in which the seller can engage in a strategy of gradual learning. On the other hand, learning is restricted by

[19] Notice that this is a variation of the mechanism proposed by Goethe described at the Introduction, where he does not commit to the probability with which he will send each reservation price. It is assumed this commitment of Goethe to construct the equilibrium below.

[20] Beccuti (2014), which can be downloaded from the web, studies the multi-period setting for an equal level of patience.

the sequential monotonicity constraint. Large discount factors not only implies that separation and some semi-separation price posting are not feasible but also that the seller cannot take advantage from learning with a non-price posting mechanism.

It can be also analyzed the case with many buyers. Bester and Strausz (2000) shows that a direct mechanism with truthful reporting is not possible in a multi-buyer case. In the same direction, Evans and Reiche (2008) proves that the revelation principle fails in the multi-buyer setting but only if at least two buyers have private information. To study an environment with more than one privately informed buyer it is necessary to consider another approach.

6 Appendix

6.1 Defining a new mechanism that changes role of messages.

Lemma: *An incentive feasible outcome* (q_r, p_r, Γ_r)*, where* Γ_r *is a direct mechanism, such that* $q_L > q_H$ *is payoff equivalent to an incentive feasible outcome* $(\hat{q}_r, \hat{p}_r, \Gamma_r)$*, with the same direct mechanism* Γ_r*, such that* $\hat{q}_H > \hat{q}_L$*.*

Proof. Suppose an incentive feasible outcome (q_r, p_r, Γ_r), where Γ_r is a direct mechanism, and with $q_L > q_H$. Since $q_H > 0$ and $q_L < 1$ by the Revelation Principle, all IC constraints hold with equality, i.e.

$$\begin{aligned}
&IC_{H,r}\text{:}\;\; u_{H,r}(h) + \delta U_{H,r-1}(p_r(h)) = u_{H,r}(l) + \delta U_{H,r-1}(p_r(l)),\\
&IC_{L,r}\text{:}\;\; u_{L,r}(l) + \delta U_{L,r-1}(p_r(l)) = u_{L,r}(h) + \delta U_{L,r-1}(p_r(h)).
\end{aligned}$$

By incentive feasibility IR and BR are satisfied,

$$\begin{aligned}
&IR_{H,r}\text{:}\;\; u_{H,r}(h) + \delta U_{H,r-1}(p_r(h)) \geq 0,\\
&IR_{L,r}\text{:}\;\; u_{L,r}(l) + \delta U_{L,r-1}(p_r(l)) \geq 0,\\
&BR_r\text{:}\;\; p_{H,r}(m_r) \sum_{j \in \Theta} p_{j,r+1} q_j(m_r) = p_{H,r+1} q_H(m_r) \quad \text{with } m_r = l, h.
\end{aligned}$$

The new outcome $(\hat{q}_r, \hat{p}_r, \Gamma_r)$ is created by renaming types such that now, $\hat{q}_H = q_L$ and $\hat{q}_L = q_H$. Hence $\hat{q}_H > \hat{q}_L$. It is straightforward to check that new constraints are all satisfied, with $IC_{H,r} = I\hat{C}_{L,r}$, $IC_{L,r} = I\hat{C}_{H,r}$, $IR_{H,r} = I\hat{R}_{L,r}$, $IR_{L,r} = I\hat{R}_{H,r}$ and $p_{H,r}(m_r) = \hat{p}_{L,r}(m_r)$.

Also, the seller remains indifferent, i.e.

$$\sum_{i \in \Theta} \sum_{m_r \in \{l,h\}} p_{i,r+1} q_i(m_r) \left[v_r(m_r) + \delta V_{r-1}(p_r(m_r))\right] = \sum_{i \in \Theta} \sum_{m_r \in \{l,h\}} \hat{p}_{i,r+1} \hat{q}_i(m_r) \left[v_r(m_r) + \delta V_{r-1}(\hat{p}_r(m_r))\right].$$

■

6.2 Proof of Remark 1

Proof. Consider a static framework, a message set $M_1 = \{\text{"}take - it\text{"}, \text{"}leave - it\text{"}\}$, and the following allocation rule

$$x_1(m_1) = \begin{cases} 1 & if \quad m_1 = take - it, \\ 0 & if \quad m_1 = leave - it, \end{cases} \quad, m_1 \in M_1.$$

Define the probabilities of observing each message with

$$\begin{aligned}
&\hat{q}_i(take - it) \equiv q_i x_1(h) + (1 - q_i) x_1(l),\\
&\hat{q}_i(leave - it) \equiv 1 - \hat{q}_i(take - it),
\end{aligned}$$

Hence, by the Revelation Principle $q_H = 1$ and $q_L = 0$ then $\hat{q}_H(take - it) = 1$, $\hat{q}_L(take - it) = x_1(l)$.

When $p_{H,2} < \theta_L/\theta_H$ the optimal direct selling mechanism has allocations $x_1(l) = 1$, then $\hat{q}_L(take - it) = 1$. Using a price $\hat{w}_1(take - it) = \theta_L$, instant payoffs under both mechanisms are equal for every player.

When $p_H \geq \theta_L/\theta_H$, $x_1(l) = 0$ and $\hat{q}_L(take - it) = 0$. The optimal direct selling mechanism has payments $w_1(h) = \theta_H$ and $w_1(l) = 0$. Using $\hat{w}_1(take - it) = w_1(h)$, instant payoffs under both mechanisms are equal for every player. Therefore, both mechanisms are payoff equivalent. ■

6.3 Proof of Lemma 2

Proof. This proof follows a similar procedure than in the static case. So, it has three steps:

Step 1: $IC_{H,2} + IR_{L,2} \Longrightarrow IR_{H,2}$,

First, notice that $u_{H,2}(m_2) + \delta U_{H,1}(p_2(m_2)) \geq u_{L,2}(m_2) + \delta U_{L,1}(p_2(m_2))$ $\forall m_2$ by $x_2(m_2)\theta_H - w_2(m_2) \geq x_2(m_2)\theta_L - w_2(m_2)$, $U_{L,1}(p_2) = 0$ $\forall p_2$ and $U_{H,1}(p_2) \geq 0$ $\forall p_2$.

Therefore, $u_{H,2}(l) + \delta U_{H,1}(p_2(l)) \geq u_{L,2}(l) + \delta U_{L,1}(p_2(l))$ and, by $IR_{L,2}$ and $IC_{H,2}$, it must be that $u_{H,2}(h) + \delta U_{H,1}(p_2(h)) \geq 0$, i.e. $IR_{H,2}$ holds.

Step 2: Optimality $\Longrightarrow IR^*_{L,2} + IC^*_{H,2}$,

Optimality means that the seller proposes an outcome (q_2, p_2, Γ_2) that maximize her profits.

Recall that, since $p_2(h) \geq p_2(l)$, $U_{H,1}(p_2(h)) \leq U_{H,1}(p_2(l))$.

Let firs prove that optimality implies $IR^*_{L,2}$.

From step 1, $u_{H,2}(h) + \delta U_{H,1}(p_2(h)) \geq u_{L,2}(l) + \delta U_{L,1}(p_2(l))$, with $U_{L,1}(p_2(l)) = 0$.

Now assume that both types start with the same payment $\hat{w}_2$, then

$$x_2(h)\theta_H - \hat{w}_2 + \delta U_{H,1}(p_2(h)) \geq x_2(l)\theta_L - \hat{w}_2 > 0.$$

In order to improve her payoffs, the seller increases the payment $\hat{w}_2$ asked to both types by some amount Δw. She continues doing that up to $x_2(l)\theta_L - \hat{w}_2 - \Delta w = 0$, i.e. $IR_{L,2}$ is binding and $w_2(l) = \hat{w}_2 + \Delta w$. Note that $IC_{H,2}$ and $IC_{L,2}$ both hold while changing $\hat{w}_2$. A larger increment in Δw violates $IR_{L,2}$.

To prove that optimality implies $IC^*_{H,2}$, notice first that continuation values are completely determined by q_H and q_L and suppose some fixed continuation values for the high-type buyer. Once the seller fixes $w_2(l)$, she continues increasing the payment for message h by $\Delta w'$ up to

$$x_2(h)\theta_H - (w_2(l) + \Delta w') + \delta U_{H,1}(p_2(h)) = x_2(l)\theta_H - w_2(l) + \delta U_{H,1}(p_2(l)).$$

During the process, she could be inducing different q_H and q_L but in such a way that continuation values do not change. At this point, the seller does not increase the payment anymore. If it were the case, the high-type buyer will send a low-type message, violating $IC_{H,2}$. As consequence, $IC_{H,2}$ is binding for those particular continuation values. For another pair of continuation values, the seller follows the same procedure, getting again $IC_{H,2}$ binding for the new continuation values. Hence, when maximizing her profits, the seller chooses some outcome (q_2, p_2, Γ_2) such that $IC_{H,2}$ is binding.

Note that $IC_{L,2}$ continues holding while the seller increases the payment $\Delta w'$.

Step 3: $IR^*_{L,2} + IC^*_{H,2} + SMC_2 \Leftrightarrow IR^*_{L,2} + IC^*_{H,2} + IC_{L,2}$

$\Leftarrow$: Operating with $IC^*_{H,2}$:

$$w_2(h) - w_2(l) = x_2(h)\theta_H - x_2(l)\theta_H + \delta\left[U_{H,1}(p_2(h)) - U_{H,1}(p_2(l))\right].$$

Plugging it into $IC_{L,2}$ and operating again:

$$x_2(h) + \delta\frac{U_{H,1}(p_2(h))}{\Delta\theta} \geq x_2(l) + \delta\frac{U_{H,1}(p_2(l))}{\Delta\theta} \text{ with equality if } q_L > 0,$$

the Sequential Monotonicity Constraint (SMC_2)

$\Rightarrow$: Starting from the SMC_2, multiplying it by $\Delta\theta$ and using $IC^*_{H,2}$ and $IR^*_{L,2}$ it is recovered $IC_{L,2}$. ■

6.4 Feasibility of candidates

Lemma: *A mechanism with SMC non-binding with no-learning is offered only when* $p_H \geq \frac{\theta_L}{\theta_H^2}\Delta\theta + \frac{\theta_L}{\theta_H}$ *and one with SMC non-binding with learning only when* $p_H \geq \theta_L/\theta_H$. *Mechanisms with SMC binding have no restrictions on the prior.*

Proof. A mechanism with SMC non-binding with no-learning requires $q_L = 0$, $\rho_{H,3} \geq \theta_L/\theta_H$ and $p_{H,2}(l) \geq \theta_L/\theta_H$. Since $q_L = 0$, $\rho_{H,3} \geq \theta_L/\theta_H$ iff $q_H \geq \frac{\theta_L}{\theta_H p_H}$ and $p_{H,2}(l) \geq \theta_L/\theta_H$ iff $q_H \leq 1 - \frac{(1-p_H)}{p_H}\frac{\theta_L}{\Delta\theta}$. Both conditions are satisfied when $p_H \geq \frac{\theta_L}{\theta_H^2}\Delta\theta + \frac{\theta_L}{\theta_H}$.

A mechanism with SMC non-binding with learning requires $q_L = 0$, $\rho_{H,3} \geq \theta_L/\theta_H$ and $p_{H,2}(h) \geq \theta_L/\theta_H > p_{H,2}(l)$. Now, again since $q_L = 0$, $q_H \geq \frac{\theta_L}{\theta_H p_H}$ and, $q_H > 1 - \frac{(1-p_H)}{p_H}\frac{\theta_L}{\Delta\theta}$. From $q_H \in [0, 1]$, first condition is satisfied only when $p_H \geq \theta_L/\theta_H$, and the second one for any $p_H < 1$.

Finally, a mechanism with SMC binding requires $q_L \neq 0$ or $q_L = 0$ and $\rho_{H,3} < \theta_L/\theta_H$. In case of learning, it also requires $p_{H,2}(h) \geq \theta_L/\theta_H$ (when $p_H < \theta_L/\theta_H$) or $p_{H,2}(l) < \theta_L/\theta_H$ (when $p_H \geq \theta_L/\theta_H$). In case of no-learning, $p_{H,2}(h) < \theta_L/\theta_H$ when $p_H < \theta_L/\theta_H$ or $p_{H,2}(l) \geq \theta_L/\theta_H$ when $p_H \geq \theta_L/\theta_H$. There is no restriction on the prior for both cases of SMC binding. ■

6.5 Candidates to be optimal mechanisms when seller and buyer are equally patient.

1. SMC binding with no-learning (SMC*+NL)
   By no-learning, $U_{H,1}(p_{H,2}(l)) - U_{H,1}(p_{H,2}(h)) = 0$. Then, the expected continuation value for the seller is equal to $V_1(p_H)$ (see discussion following Definition 2). By SMC binding, $x_2(l) \neq 0$. From (10) and no-learning it must be that $x_2(l) = 1$. Substituting continuation values and allocations at (**??**) and after some simplifications results in the seller's maximum payoffs equal to

   $$\theta_L + \delta \max\{p_H\theta_H, \theta_L\}.$$

   The seller is indifferent among any pair (q_L, q_H) such that there is SMC binding with no-learning. Hence, assume $q_L = q_H \neq 0$ (i.e. $p_{H,2}(h) = p_{H,2}(l) = p_H$) without a loss of generality.

2. SMC binding with learning (SMC*+L)
   Now $U_{H,1}(p_{H,2}(l)) - U_{H,1}(p_{H,2}(h)) \neq 0$ by learning. Because $p_{H,2}(h) \geq \theta_L/\theta_H > p_{H,2}(l)$, $U_{H,1}(p_{H,2}(l)) - U_{H,1}(p_{H,2}(h)) = \Delta\theta$ by (6) and, $V_1(p_{H,2}(h)) = p_{H,2}(h)\theta_H$ and $V_1(p_{H,2}(l)) = \theta_L$ by (**??**). By SMC binding $x_2(l) \neq 0$, and jointly with learning, $x_2(l) = 1 - \delta$. Substituting allocations and continuation values at (**??**) and after some simplifications, the seller chooses (q_L, q_H) to maximize

   $$\theta_L + \delta\rho_{H,3}p_{H,2}(h)\theta_H.$$

   According to Bayes' rule $\rho_{H,3}p_{H,2}(h) = p_H q_H$. Because $q_H \leq 1$, a mechanism with SMC binding with learning is weakly dominated by a mechanism under case 1 for any prior.

3. SMC non-binding with learning (SMC+L)
   Now $U_{H,1}(p_{H,2}(l)) - U_{H,1}(p_{H,2}(h)) = \Delta\theta$ by learning and $x_2(l) = 0$ by SMC non-binding. The allocation $x_2(l) = 0$ implies that $q_L = 0$ and $\rho_{H,3} \geq \theta_L/\theta_H$, i.e. $q_H \geq \frac{\theta_L}{\theta_H p_H}$. This last requirement jointly with $p_{H,2}(h) \geq \theta_L/\theta_H > p_{H,2}(l)$ (by learning) implies $p_H \geq \theta_L/\theta_H$ (see Lemma 4 in the Appendix). From (**??**), $V_1(p_{H,2}(h)) = p_{H,2}(h)\theta_H$ and $V_1(p_{H,2}(l)) = \theta_L$. Therefore, the seller chooses q_H to maximize

   $$\begin{aligned}&\rho_{H,3}\theta_H + \delta\theta_L,\\ &\text{subject to } q_L = 0,\ \rho_{H,3} \geq \theta_L/\theta_H,\ p_{H,2}(l) < \theta_L/\theta_H,\end{aligned}$$

   getting

   $$p_H\theta_H + \delta\theta_L.$$

   when $q_H = 1$.

4. SMC non-binding with no-learning (SMC+NL)
   no-learning means $U_{H,1}(p_{H,2}(l)) - U_{H,1}(p_{H,2}(h)) = 0$ with expected continuation value for the seller equals to $V_1(p_H)$ (see above). SMC non-binding means $x_2(l) = 0$. The necessary conditions for the optimum of the first problem implies that $q_L = 0$ and $\rho_{H,3} \geq \theta_L/\theta_H$, i.e. $q_H \geq \frac{\theta_L}{\theta_H p_H}$. Additionally, by no-learning, it must be that $p_{H,2}(l) \geq \theta_L/\theta_H$.[21] To satisfy previous requirements it is necessary that $p_H \geq p^*$ (where $p^* = \frac{\theta_L}{\theta_H^2}\Delta\theta + \frac{\theta_L}{\theta_H}$, see Lemma 4 in the Appendix). After substitutions at (**??**) and

[21] $q_L = 0$ gives $p_{H,2}(h) = 1$ by BR_2. Then, by *non-learning*, it must be that $p_{H,2}(h) \geq p_{H,2}(l) \geq \frac{\theta_L}{\theta_H}$.

simplifications, the seller chooses q_H to maximize

$$\rho_{H,3}\theta_H + \delta p_H \theta_H,$$
$$\text{subject to } q_L = 0,\ \rho_{H,3} \geq \theta_L/\theta_H,\ p_{H,2}(l) \geq \theta_L/\theta_H,$$

getting

$$\frac{p_H\theta_H - \theta_L}{\Delta\theta}\theta_H + \delta p_H \theta_H,$$

with $q_H = \frac{p_H\theta_H - \theta_L}{p_H \Delta\theta}$.

6.6 Proof of Corollary 1

Proof. Consider for period $r = 2$ a message set $M_2 = \{"take-it", "leave-it"\}$, and the following allocation rule

$$x_2(m_2) = \begin{cases} 1 & if \quad m_2 = take-it, \\ 0 & if \quad m_2 = leave-it, \end{cases},\ m_2 \in M_2.$$

Define the probabilities of observing each message with

$$\hat{q}_i(take-it) \equiv q_i x_2(h) + (1-q_i)x_2(l),$$
$$\hat{q}_i(leave-it) \equiv 1 - \hat{q}_i(take-it).$$

Posteriors $\hat{p}_{i,2}(take-it)$ and $\hat{p}_{i,2}(leave-it)$ are given by Baye's rule.

When $p_H < \theta_L/\theta_H$ the optimal direct selling mechanism has allocations $x_2(h) = x_2(l) = 1$, hence $\hat{q}_H(take-it) = 1$, $\hat{q}_L(take-it) = 1$ and $\hat{p}_{H,2}(take-it) = p_H$. It follows that continuation values with the price posting are equal than under the direct mechanisms, i.e., $U_{i,1}(\hat{p}_2(take-it)) = U_{i,1}(p_{H,2}(h))$ for both types and $V_1(\hat{p}_2(take-it)) = V_1(p_{H,2}(h))$. Using a price $\hat{w}_2(take-it) = \theta_L$, instant payoffs under both mechanisms are also equal for every player.

When $p_H \geq \theta_L/\theta_H$ the optimal direct selling mechanism has payments $w_2(h) = \theta_H$ and $w_2(l) = 0$, or $w_2(h) = \theta_H - \delta\Delta\theta$ and $w_2(l) = 0$, with allocations $x_2(h) = 1$ and $x_2(l) = 0$. It follows that: $\hat{q}_H(take-it) = q_H$, $\hat{q}_L(take-it) = q_L$, $\hat{p}_{H,2}(take-it) = p_{H,2}(h)$ and $\hat{p}_{H,2}(leave-it) = p_{H,2}(l)$. Again, continuation values are equal for both mechanisms, i.e., $U_{i,1}(\hat{p}_2(take-it)) = U_{i,1}(p_2(h))$, $U_{i,1}(\hat{p}_2(leave-it)) = U_{i,1}(p_2(l))$, $V_1(\hat{p}_2(take-it)) = V_1(p_2(h))$ and $V_1(\hat{p}_2(leave-it)) = V_1(p_2(l))$. Using $\hat{w}_2(take-it) = w_2(h)$, instant payoffs under both mechanisms are also equal for every player.

Then, for every prior, it is possible to implement an outcome $(\hat{q}_2, \hat{p}_2, \hat{\Gamma}_2)$, where $\hat{\Gamma}_2$ is a price posting mechanism, which is payoff equivalent to the incentive efficient outcome (q_2, p_2, Γ_2) that solves (7) ■

6.7 Proof of Proposition 2

Proof. Since seller's payoffs under the non-price posting mechanism are bounded above by pooling (SMC*+NL) for any prior when $\beta < \delta$, the following proof focus on $\beta > \delta$.

The structure of the proof is the following: First, it proves that a price posting mechanism is always optimal when $p_H \geq \theta_L/\theta_H$. Next, it proves that SMC*+L is the optimal mechanism for certain priors when $p_H < \theta_L/\theta_H$. Finally, it proves that, in this last case, there is not a price posting mechanism payoff equivalent to SMC*+L.

Consider $p_H \geq \theta_L/\theta_H$. Seller's payoffs under SMC+NL are larger than those under SMC+L when $p_H \geq \frac{\theta_L(\theta_H + \beta\Delta\theta)}{[\theta_L\theta_H + \Delta\theta(\delta\theta_H + (\beta-\delta)\theta_L)]}$ and the opposite otherwise (provided that both are defined for the prior). On the other hand, SMC+L gives larger seller's payoffs than any of the mechanisms with SMC*. Hence, price posting is optimal with such a prior.

Consider now $p_H < \theta_L/\theta_H$. For these priors SMC+NL and SMC+L are not defined. On the other hand, SMC*+L gives larger payoffs to the seller than SMC*+NL when

$$\beta p_H \Delta\theta + \delta p_H \theta_L > \delta\theta_L.$$

Suppose $\beta = \delta + \varepsilon$. After operating, previous inequality holds if

$$p_H > \frac{\delta\theta_L}{\delta\theta_H + \varepsilon\Delta\theta}.$$

Since $\delta\theta_H + \varepsilon\Delta\theta > \delta\theta_H$ for all $\varepsilon > 0$, then there exist $p_H \in \left(\frac{\delta\theta_L}{\delta\theta_H+\varepsilon\Delta\theta}, \frac{\theta_L}{\theta_H}\right)$ such that payoffs under SMC*+L are larger than payoffs under SMC*+NL.

It remains to show that there is not a price posting mechanism payoff equivalent to the SMC*+L when the seller finds optimal to implement it. Notice that SMC*+L implies $q_H = 1$, $q_L = 0$. Following definitions in the proof of Corollary 1, if SMC*+L can be implemented by a price posting, it must be that $\hat{q}_H(take-it) = x_2(h)$ (i.e., $\hat{q}_H(take-it) = 1$) and $\hat{q}_L(take-it) = x_2(l)$ (i.e., $\hat{q}_L(take-it) = 1-\delta$).

The seller can offer a price posting mechanisms with a price lower than θ_L, equal to θ_L or larger than θ_L.

Suppose a mechanism with a price lower to θ_L and payoff equivalent to SMC*+L. No matter whether the seller learns or not with this price posting, low-type buyer makes zero payoff in the last period while he gets positive payoff in the first period sending "$take-it$" and zero with "$leave-it$". This is implies that low-type strictly prefers a message "$take-it$" than "$leave-it$", i.e., $\hat{q}_L(take-it) = 1$ which is a contradiction for any $\delta \neq 0$.

Suppose now a mechanism with a price larger than θ_L and payoff equivalent to SMC*+L. Now, no matter posteriors, low-type buyer strictly prefers sending "$leave-it$", otherwise he makes negative profits. This is, $\hat{q}_L(take-it) = 0$, contradiction (δ cannot be 1 when $\delta < \beta$).

Finally, consider the case of a price posting mechanism with a price larger equal to θ_L and payoff equivalent to SMC*+L. If the seller does not learn, she asks for a price equal to the low-type buyer's valuation in the second period (recall $p_H < \theta_L/\theta_H$), making $\theta_L + \beta\theta_L$ which is payoff equivalent to SMC*+L only when $p_H = \frac{\delta\theta_L}{\delta\theta_H+\varepsilon\Delta\theta}$. If she learns, in the second period she will propose a price θ_H when she observes "$take-it$", and θ_L when she observes "$leave-it$". Hence, high-type buyer strictly prefers "$take-it$", i.e., $\hat{q}_H(take-it) = 1$ and $\hat{p}_{H,2}(leave-it) = 0$. On the other hand, low-type buyer is indifferent between both messages. Assume that $\hat{q}_L(take-it) = 1-\delta$ (as it is required). In this case, the seller gets

$$\begin{aligned} V_2 &= [p_H + (1-p_H)(1-\delta)]\,[\hat{w}_2(take-it) + \beta V_1(\hat{p}_2(take-it))] \\ &\quad + [1 - p_H - (1-p_H)(1-\delta)]\,[\beta V_1(\hat{p}_2(leave-it))] \\ &= (p_H + (1-p_H)(1-\delta))\,\theta_L + \beta p_H\theta_H + \beta\delta(1-p_H)\theta_L, \end{aligned}$$

which is lower than seller's payoffs under SMC*+L. Hence, this price posting is not payoffs equivalent to SMC*+L when is optimal to the seller to propose it.

When $\beta < \delta$, seller's payoffs under the non-price posting mechanism are bounded above by pooling (SMC*+NL). ■

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